

# Full Line Constants Feature

## Overview

This document describes the Full Line Constants feature in PLS-CADD, available in v15.42 and later, which calculates all three symmetrical components (zero, positive, and negative sequence values) of the line impedance. It accounts for ground wires, ground return, and mutual coupling between circuits. Note that the Simple Line Constants feature, which was available in earlier versions, only calculates the positive sequence and does not factor in ground returns or mutual coupling.

## PLS-CADD Advantage

The main advantage of calculating line constants with PLS-CADD is that, in addition to already having the full layout of the structure locations and the attachment positions, it has a detailed model of the wire positions and height throughout all spans for specified weather cases. Instead of using just a single set of spacings between wires for a span or section of line, PLS-CADD evaluates the height and spacing between wires at several points throughout each span. This provides a more accurate representation of the height and separation between wires particularly for spans over uneven ground and situations where the wire arrangement changes within a span such as when rolling between horizontal and vertical configurations. In addition, the ability to easily change the weather case and wire positions modeled for that case, allows you to easily evaluate the possible effects of those different conditions (e.g. changes in sag and blow-out).

Starting with v16.05, PLS-CADD supports calculating the mutual impedance between adjacent spans and different earth resistivity for different spans in a project.

## User Input

This feature requires a PLS-CADD project containing at least one line with structures placed and sections strung, along with an appropriate weather case and a ground TIN beneath the spans of interest. Circuit and phase labels also need to be assigned to any wires that you want to analyze (see the [technical note](#) or [video](#) on circuit labeling). You specify which spans to analyze by structure and which circuit labels to analyze as conductors and which circuit labels to analyze as ground wires. You may choose to omit any circuit labels from the analysis (which means they would not factor in for mutual coupling). There are also options to adjust the frequency (defaults to 60hz), and earth resistivity (defaults to 100 ohm\*m). You specify the maximum distance and relative angle for nearby spans to be considered adjacent and parallel for mutual coupling impedance. You can also enable a feature that will calculate a different earth resistivity per span based on earth resistivity values assigned to structures.

## Phase Impedance Matrix with Grounds

The first step in the calculation is to generate the phase impedance matrix with grounds for every user selected span. The dimensions for this matrix are determined by the total number of wires within the span that is being analyzed including conductors and ground wires. For example, on a double 3-phase circuit with two ground wires, this matrix will be 8x8. If there are adjacent spans, all wires from those spans are included as well to account for the mutual coupling effects. Each  $Z_{mn}$  term within the matrix is calculated using Carson's equations which take into account the relative wire positions, the wire's height above ground, and the mirror image wire position below ground which represents the ground return current. As noted above, the program will calculate each  $Z_{mn}$  term at several positions along the span and average them together to provide a more representative value of the entire span.

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_m \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1n} \\ Z_{21} & Z_{22} & \cdots & Z_{2n} \\ \vdots & \vdots & & \vdots \\ Z_{m1} & Z_{m2} & \cdots & Z_{mn} \end{bmatrix} \cdot \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_m \end{bmatrix}$$

$$[V_{abc}] = [Z_{abc}] \cdot [I_{abc}]$$

Carson's Equations (from EPRI Redbook section 2.5.2):

$$\text{when } m=n, \quad z_{mm} = R_m + 4\pi \cdot 10^{-7} f \left( j \cdot \ln \left( \frac{D_{mm'}}{GMR_m} \right) + 2(P+jQ) \right) \quad \Omega/m$$

$$\text{when } m \neq n, \quad z_{mn} = 4\pi \cdot 10^{-7} f \left( j \cdot \ln \left( \frac{D_{mn'}}{D_{mn}} \right) + 2(P+jQ) \right) \quad \Omega/m$$

$$P = \frac{\pi}{8} - \frac{1}{3\sqrt{2}} k \cdot \cos \theta + \frac{k^2}{16} \cos 2\theta \left( 0.6728 + \ln \frac{2}{k} \right) + \frac{k^2}{16} \theta \sin 2\theta + \frac{k^3 \cos 3\theta}{45\sqrt{2}} - \frac{\pi k^4 \cos 4\theta}{1536}$$

$$Q = -0.0386 + \frac{1}{2} \ln \left( \frac{2}{k} \right) + \frac{1}{3\sqrt{2}} k \cos \theta - \frac{\pi k^2}{64} \cos 2\theta + \frac{k^3 \cos 3\theta}{45\sqrt{2}} - \frac{k^4 \theta}{384} \sin 4\theta - \frac{k^4 \cos 4\theta}{384} \left( \ln \frac{2}{k} + 1.0895 \right)$$

$$\text{when } m=n, \quad k = 2.81 \times 10^{-3} D_{mm'} \sqrt{f/\rho} \quad \text{and} \quad \theta = 0$$

$$\text{when } m \neq n, \quad k = 2.81 \times 10^{-3} D_{mn'} \sqrt{f/\rho} \quad \text{and} \quad \theta = \arcsin(H_{mn}/D_{mn'})$$

where

$R_m$  = AC resistance of conductor  $m$  in  $\Omega/m$

$f$  = user specified frequency in Hertz

$GMR_m$  = geometric mean radius for conductor  $m$

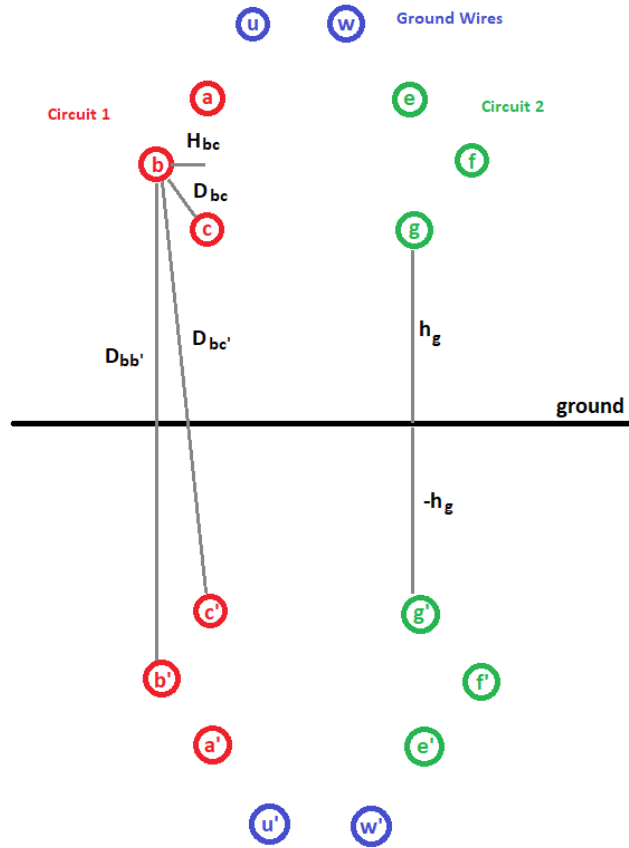
$D_{mn}$  = distance between conductors  $m$  and  $n$

$D_{mm'}, D_{mn'}$  = distance between conductor  $m$  and image conductor  $m'$  or  $n'$

$\rho$  = user specified resistivity of earth in  $\Omega m$

$H_{mn}$  = horizontal distance between conductors  $m$  and  $n$

Wire Position Diagram with ground return mirror image wires:



## Reduced Phase Impedance Matrix

The next step is to reduce the phase impedance matrix by using Kron Reduction to factor out each ground wire. This reduced matrix becomes the final phase impedance matrix associated with each span. If the span contains just one circuit, that matrix is a simple 3x3 of the phase self impedances and mutual impedances. If the span contains 2 or more circuits, the matrix is a 6x6 or larger and contains the 3x3 phase impedance matrix for each circuit plus additional 3x3 matrices for the mutual impedances between circuits.

$$Z_{mn}^{new} = Z_{mn}^{old} - \frac{Z_{mw} \cdot Z_{wn}}{Z_{ww}}$$

$$\begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} & Z_{ae} & Z_{af} & Z_{ag} \\ Z_{ba} & Z_{bb} & Z_{bc} & Z_{be} & Z_{bf} & Z_{bg} \\ Z_{ca} & Z_{cb} & Z_{cc} & Z_{ce} & Z_{cf} & Z_{cg} \\ Z_{ea} & Z_{eb} & Z_{ec} & Z_{ee} & Z_{ef} & Z_{eg} \\ Z_{fa} & Z_{fb} & Z_{fc} & Z_{fe} & Z_{ff} & Z_{fg} \\ Z_{ga} & Z_{gb} & Z_{gc} & Z_{ge} & Z_{gf} & Z_{gg} \end{bmatrix} = \begin{bmatrix} [Z_{Ckt1}] & [Z_{Ckt1Ckt2}] \\ [Z_{Ckt2Ckt1}] & [Z_{Ckt2}] \end{bmatrix} \Omega/m$$

## Symmetrical Components

Each 3x3 phase impedance matrix is then converted to a sequence impedance matrix using these equations:

$$[Z_{012}] = [A_s]^{-1} \cdot [Z_{abc}] \cdot [A_s]$$

$$\begin{bmatrix} z_{00} & z_{01} & z_{02} \\ z_{10} & z_{11} & z_{12} \\ z_{20} & z_{21} & z_{22} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a_s & a_s^2 \\ 1 & a_s^2 & a_s \end{bmatrix} \cdot \begin{bmatrix} z_{aa} & z_{ab} & z_{ac} \\ z_{ba} & z_{bb} & z_{bc} \\ z_{ca} & z_{cb} & z_{cc} \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 1 \\ 1 & a_s^2 & a_s \\ 1 & a_s & a_s^2 \end{bmatrix} \Omega/m$$

$$a_s = -0.5 + 0.866j$$

$$a_s^2 = -0.5 - 0.866j$$

$$a_s^3 = 1$$

In the above matrix, the  $z_{00}$  term is the zero sequence impedance, the  $z_{11}$  term is the positive sequence impedance, and the  $z_{22}$  term is the negative sequence impedance. The other terms are the mutual coupling between sequences.

## Potential Coefficient Matrix with Grounds

To determine the capacitance and susceptance, PLS-CADD first calculates the potential coefficient matrix with grounds. The dimensions of this matrix are the same as the phase impedance matrix with grounds described above with a row and column for every conductor and ground wire. Each potential coefficient is calculated as shown in these equations:

$$[V_{abc}] = [P_{abc}] \cdot [Q_{abc}]$$

$$\begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_m \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{2n} \\ p_{21} & p_{22} & \cdots & p_{2n} \\ \vdots & \vdots & & \vdots \\ p_{m1} & p_{m2} & \cdots & p_{mn} \end{bmatrix} \cdot \begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_m \end{bmatrix}$$

$$\text{when } m=n, \quad p_{mm} = 1.7975 \times 10^{10} \cdot \ln\left(\frac{D_{mm'}}{r_m}\right) \text{ meter / farad}$$

$$\text{when } m \neq n, \quad p_{mn} = 1.7975 \times 10^{10} \cdot \ln\left(\frac{D_{mn'}}{D_{mn}}\right) \text{ meter / farad}$$

where

$r_m$  = radius of each conductor

$D_{mn}$  = distance between conductors  $m$  and  $n$

$D_{mm'}, D_{mn'}$  = distance between conductor  $m$  and image conductor  $m'$  or  $n'$

## Reduced Potential Coefficient Matrix

As with the phase impedance matrix, the ground wire rows and columns in the potential coefficient matrix are factored out using Kron Reduction as shown above. This results in a matrix with dimensions equal to the total number of conductor wires in the span.

## Capacitance and Susceptance Matrices

The potential coefficient matrix is inverted to determine the capacitance matrix. Then that matrix is multiplied by the radial frequency to calculate the susceptance matrix.

$$[V_{abc}] = \frac{1}{j\omega} [P_{abc}] \cdot [I_{abc}]$$

$$[C] = [P]^{-1}$$

$$\begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_m \end{bmatrix} = j\omega \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ c_{21} & c_{22} & \cdots & c_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ c_{m1} & c_{m2} & \cdots & c_{mn} \end{bmatrix} \cdot \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_m \end{bmatrix}$$

$$\omega = 2\pi f$$

## Final Calculations

PLS-CADD knows the length of each wire in every span and can multiply either the phase or sequence impedance values by those lengths to get the actual span impedance for each wire. Note that PLS-CADD calculates wire lengths between the wire attach points on the insulators and accounts for that wire's sag under the specified weather case. PLS-CADD uses the circuit and phase labels to sum the individual span results into total circuit results for the specified structure ranges. This includes accounting for phase transpositions between spans. The circuit voltage and the specified System Base Power are used to calculate the Per-Unit and Reactive Power results.

## Report Output

The report generated by this command contains a few standard tables plus several optional tables for more detailed or intermediate calculation results. The Section Data table shows the conditions for all wires being analyzed including the calculated effective resistance, radius, and GMR based on the modeled weather case temperature and if the wire is bundled. The Circuit by Span Results table shows the span length and the average and total impedance and susceptance for zero and positive sequences for every span. The Circuit Results table summarizes those results for each circuit and includes Per-Unit, Magnitude and Phase, and Reactive Power results for both zero and positive sequence. Starting with v15.60 this report also calculates the symmetrical components of the mutual phase impedance between circuits and includes tables showing the average and total zero sequence mutual impedance between circuits per span and summarized for each circuit. The optional tables include the full matrix results (both final and intermediate) for each circuit and for every span. There is also an option to include the average modeled wire positions for each span used for these calculations.

## Mutual Coupling Between Circuits

Prior to v16.05, this feature only calculated the mutual coupling between circuits if those circuits are on the same span (meaning they have the same back and ahead structures). Starting with v16.05, PLS-CADD accounts for mutual coupling between circuits within a user specified distance at angle (to determine parallel) whether or not they have the same structures. This includes circumstances where the neighboring circuits are only parallel for a portion of a span.

## References

The two primary references used as guidance for equations and approach are:

- "EPRI AC Transmission Line Reference Book--200kV and Above 3rd Ed." Section 2.5.2
- "Distribution System Modeling and Analysis 3rd Ed." by William H Kersting; Chapter 4